

How Soon to Brake and How Hard to Brake: Unified Analysis of the Envelope of Opportunity for Rear-End Collision Warnings

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Abstract

Rear-end collision warning systems are both an available and an evolving technology. The goal of these systems is to alert the driver of a dangerous situation, with sufficient time to take evasive action and avoid a collision. Their development, analysis, and evaluation requires considering the *envelope of opportunity* available to a driver—once an alert is issued—to avoid a collision.

This paper simplifies the envelope of opportunity analysis and presentation. Previous work in this area focuses on establishing the relevant kinematic equations of motion and obtaining case-specific plots of the envelope of opportunity. We have plotted the envelope of opportunity using the reaction time and the *reciprocal* of the following vehicle acceleration. Plotting the envelope in this manner readily accommodates the limiting cases of infinite following vehicle deceleration and zero reaction time and the envelope of opportunity is, in most cases, a *straight line*, allowing for direct generation of the envelope and insight into the effect of initial kinematic variables on the envelope.

Nomenclature (SI units)

a acceleration

F subscript for following vehicle

L subscript for lead vehicle

L/F subscript for lead relative to following vehicle

R, \dot{R} range, range rate

t_f final time

t_r reaction time

v velocity (speed)

x displacement

1 INTRODUCTION

Rear-end collisions comprise approximately 28 percent of the total number of crashes in the USA, according to National Highway Traffic Safety Administration data. Countermeasures to reduce the quantity and severity of rear-end collisions constitute an important component of the US Department of Transportation *Intelligent Vehicle Initiative* program [10]. The National Highway Traffic Safety Administration has a cooperative agreement with General Motors to develop an *Automotive Collision Avoidance System* (ACAS), which is scheduled for pilot and field operational testing in 2001–2002 [7]. This system will provide visual, auditory, and (perhaps) haptic warnings to a driver to warn of an impending rear-end collision. In simple terms: the system will sense when a following vehicle is following a lead vehicle, determine when the range and range rate between the vehicles constitute a hazardous condition, and, as needed, issue an appropriate warning to alert the following vehicle driver.

The independent evaluation of countermeasures plays an important role in their design and path to market. Such evaluations assess the performance of new technologies and assist federal transportation agencies in developing policies for their use. As countermeasures and other innovations impact public safety, an impartial and thorough evaluation is essential. The GM collision avoidance evaluation will address issues such as ACAS safety benefits, driver acceptance, ACAS performance and capability, and ACAS deployment potential and price. The timing of a rear-end collision warning impacts all of these issues. As such, a ready means to assess the suitability of a warning will facilitate both the design and the evaluation of a warning algorithm.

2 BACKGROUND

In terms of evaluating the timing of a warning algorithm, there are two key variables: reaction time and deceler-

ation. These variables describe how quickly and how strongly a driver responds to a collision warning alert. This paper focuses on determining the envelope of opportunity available to a driver, that is, when and how a driver *must* respond to an alert to avoid a collision.

2.1 Principal Variables and Central Question

Simple second-order polynomials suffice to describe the rectilinear kinematics between following and lead vehicles traveling along straight roadways. Thus, the equations describing the velocity and displacement of two vehicles are well established. Assigning the origin to the location of the following vehicle at $t = 0$, the following hold:

$$\begin{aligned} v_L(t) &= v_L(0) + \int_0^t a_L dt \\ x_L(t) &= R(0) + \int_0^t v_L(t) dt \\ v_F(t) &= \begin{cases} v_F(0) & \text{if } 0 \leq t \leq t_r, \\ v_F(0) + \int_{t_r}^t a_F dt & \text{if } t > t_r \end{cases} \\ x_F(t) &= \int_0^t v_F(t) dt, \\ R(t) &= x_L(t) - x_F(t) \\ \dot{R}(t) &= v_L(t) - v_F(t) \end{aligned}$$

In the sequel the argument (0) to denote the initial condition will be omitted, where it is to be understood the $R \doteq R(0)$, $v_F \doteq v_F(0)$, and so on. Furthermore, lead vehicle deceleration (if any) begins at $t = 0$. Following vehicle deceleration begins at $t = t_r$.

Two scenarios are of interest. In the first, a following vehicle approaches a lead vehicle moving at a constant speed. The question of interest is

Given a delay between a driver observing and reacting to a slower lead vehicle, what is the minimum level of following vehicle deceleration required to avoid a rear-end collision?

In the second scenario a lead vehicle decelerates at time $t = 0$, i.e., $a_L(0) < 0$, and $v_L(0)$ may be less than $v_F(0)$. The question of interest is

Given a delay between lead vehicle deceleration and following vehicle driver response, what is the minimum level of deceleration required to avoid a rear-end collision?

2.2 Previous Work

Several works employ range-range-rate (R, \dot{R}) plots to illustrate the relative motion between a following and lead vehicle. While useful in some contexts, e.g., the development of headway control algorithms [1], these plots may be less useful in the envelope of opportunity analysis in this paper. The lack of an explicit time parameter in the plots may make it difficult to determine the behavior of the individual vehicles. More significantly, the plots do not lend themselves to serving as a basis for developing analytical expressions for required reaction times and braking levels, the focus of this paper.

An *Intelligent Vehicle Initiative* evaluation context was recently presented [3]. The presentation provided various plots with axes of “time from beginning of event” versus subject vehicle deceleration. With these axes, each combination of initial velocity (assumed equal for following and lead vehicles), lead vehicle deceleration, and initial range produces a specific parabolic curve. Points lying beneath the curve correspond to a combination of a sufficient reaction time and deceleration. Such points indicate that for the given set of initial conditions, the following vehicle will not impact the slower (and slowing) lead vehicle. This work and the results provided in this paper share a common context, with the former providing essential background to the current development.

Collision warning algorithms for three scenarios are developed in [5]. The first scenario in the cited paper corresponds to a warning issued for a stopped lead vehicle. The second and third scenarios correspond to a warning issued while the lead vehicle is moving. For the second scenario the lead vehicle stops before the following vehicle stops; the opposite holds for the third scenario. The paper provides various plots with axes of time-headway versus initial velocity. Under the assumption of a following vehicle deceleration of 0.75 g, curves corresponding to various levels of lead vehicle deceleration are plotted. The curves themselves represent the time-headway required to avoid a collision.

2.3 Principal Results

The analyses in the cited works and the plots therein are useful for evaluating a given set of initial conditions and given scenario. This paper extends these works by:

- identifying limiting cases on the plots, e.g., minimum deceleration levels required for instant reaction times and maximum reaction times allowed for infinite decelerations
- using symbolic asymptotes, so that the effects of changing initial conditions or deceleration levels are readily apparent

- showing in a single plot the two envelopes of opportunity that result when a lead vehicle decelerates
- most significantly, showing that for three common rear-end crash scenarios, a plot of the required reaction time versus the *reciprocal* of the required following vehicle acceleration is a *straight line*.

This has been achieved by (1) depicting the scenarios and their initial conditions on acceleration, velocity, and displacement diagrams, rather than solely manipulating symbolic equations, and employing a geometric approach to analyzing absolute and relative displacements, and (2) solving for reaction time as a function of the initial conditions and $1/a_F$, which readily accommodates the limiting case of $a_F \rightarrow -\infty$, providing the time-to-collision variable on the plot.

2.4 Overview

The outline of the remainder of this paper is as follows. The next section develops analytical relations between the reaction time and required deceleration, for a constant speed and a decelerating lead vehicle. In Section 4 the utility of the relations is demonstrated by plotting and analyzing the performance of a published collision avoidance algorithm [5] and a hypothetical rear-end collision avoidance algorithm. The paper closes with comments on the new relations and directions for future work.

3 DEVELOPMENT OF ANALYTICAL RELATIONS

A standard approach to formulating rectilinear kinematics problems entails graphical integration of acceleration and velocity [2]. This approach often simplifies motion analysis and provides a better understanding of the motion. This understanding is particularly evident in the case considered in this paper: a following vehicle avoiding a collision with a lead vehicle.

3.1 Constant Speed Lead Vehicle

We begin by considering the case of a following vehicle approaching a lead vehicle that is traveling at a constant speed, which includes a stopped lead vehicle. The following vehicle initially approaches at a constant speed, and later decelerates at time t_r at some $a_F < 0$. For a given set of initial conditions, the relation between the reaction time and the required deceleration is of interest.

A graphical depiction of this scenario's kinematics is shown in Figure 1. In the relative coordinates shown,

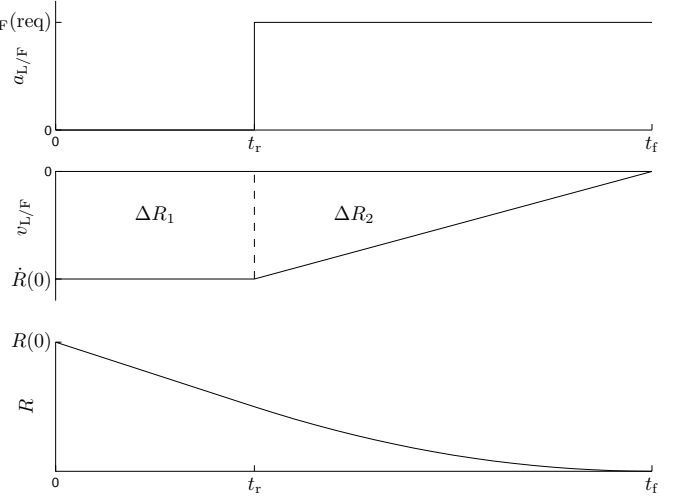


Figure 1: Relative acceleration, velocity, and displacement for constant speed lead vehicle

the negative acceleration at time t_r results in a positive relative acceleration, i.e.

$$a_{L/F}(t) = a_L(t) - a_F(t) = -a_F(t) > 0, \text{ for } t \geq t_r.$$

Prior to the reaction time, the relative velocity, $v_{L/F} = \dot{R}$, is negative and constant. After the reaction time, this variable increases to a value of zero, corresponding to the following and lead vehicles having the same speed. The relative displacement (range, R) has a constant negative slope during the prereaction interval, and the slope becomes zero as the following vehicle decelerates.

Two regions are indicated in the relative velocity graph, the middle plot in Figure 1. The area ($\int V dt$) of these regions corresponds to the change in range. The first area, ΔR_1 , is the change in range prior to the reaction time; the second, ΔR_2 , is the change after the reaction time. These changes equal

$$\Delta R_1 = \dot{R} t_r < 0 \quad (1a)$$

$$\Delta R_2 = \frac{1}{2} \dot{R} \frac{-\dot{R}}{a_{L/F}} < 0, \quad (1b)$$

$$(1c)$$

where $a_{L/F}$ actually denotes the *required* acceleration at $t = t_r$. As indicated, we seek a relationship between t_r and $a_F = -a_{L/F}$, (as $a_L = 0$), such that a combination of these two is sufficient to avoid a collision, i.e., $\dot{R} = 0$ at the instant when $R = 0$. The following equality, obtained by setting the final range equal to zero, facilitates the derivation of this relationship:

$$R + \Delta R_1 + \Delta R_2 = R + \dot{R} t_r + \frac{\dot{R}^2}{2a_F} = 0.$$

Solving for t_r results in

$$t_r = -\frac{\dot{R}}{2} \frac{1}{a_F} - \frac{R}{\dot{R}}. \quad (2)$$

Two limits result from this relation:

$$\lim_{a_F \rightarrow -\infty} t_r = -\frac{R}{\dot{R}} \quad (3a)$$

$$\lim_{t_r \rightarrow 0} \frac{1}{a_F} = -\frac{2R}{\dot{R}^2} \quad (3b)$$

A plot of the relation between t_r and $1/a_F$ for a lead vehicle traveling at a constant speed is shown in Figure 2, where a range of 54 m and a range rate of -18 m/s have been used to generate the line. The plot in Fig-

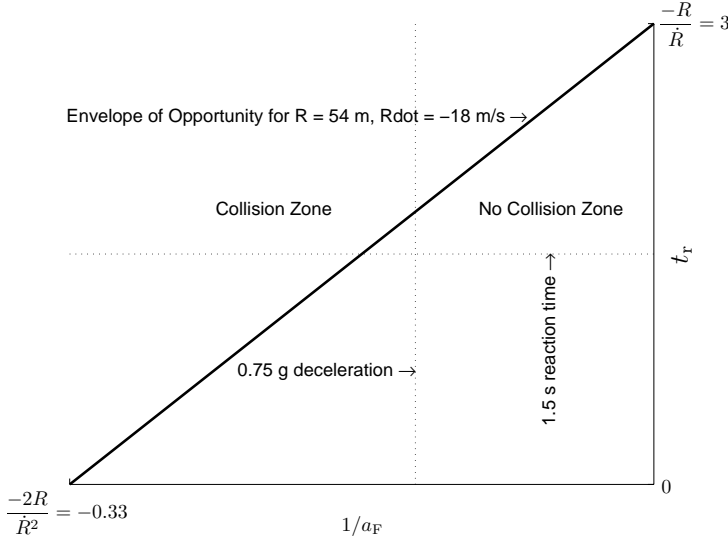


Figure 2: Envelope of opportunity for constant-speed lead vehicle

ure 2 has a direct interpretation. A straight line drawn between $-R/\dot{R} = 3$ on the y -axis and $-2R/\dot{R}^2 = -0.33$ on the x -axis provides the *envelope of opportunity*. For a lead vehicle with zero acceleration and an alert issued at range of 54 m (a 3-second headway), any combination of following vehicle reaction time and deceleration that produces a point *beneath* the straight line will avoid a collision; any combination producing a point *above* the line will lead to a collision. To put the envelope of opportunity into perspective, the figure also includes a horizontal line depicting a 1.5 second reaction time and a vertical line depicting a 0.75 g deceleration. The reaction time shown is a reasonable value for a large population segment; the deceleration level is close to a maximum value.

3.2 Decelerating Lead Vehicle

This section examines the case in which the following and lead vehicles are traveling at constant speeds and

the lead vehicle decelerates. To avoid a collision, a following vehicle must decelerate. If no collision occurs, this deceleration will eventually stop the following vehicle. Two cases will be derived. (1) Should the lead vehicle stop at a time before the following vehicle stops, a “terminal displacement” envelope of opportunity analysis will be needed. In this case, only the final relative displacement between the vehicles determines if a collision will occur. (2) Should the following vehicle stop at a time before the lead vehicle stops, a “minimum separation” envelope of opportunity analysis will be needed. In this case, the time at which the following and lead vehicles are closest determines if a collision will occur.

Figure 3 illustrates two cases of following and lead vehicle decelerating and illustrates why two analyses are needed. In the upper half of the figure, the lead vehi-

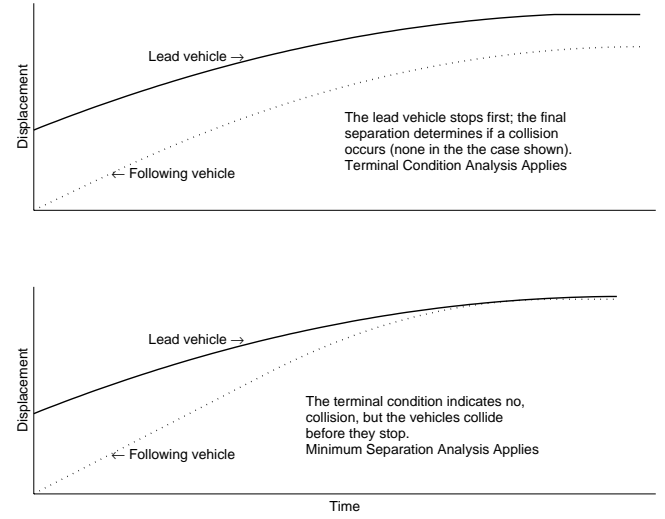


Figure 3: Displacements for terminal condition and minimum separation cases

cle stops first, which can be deduced from the constant value of the displacement curve. When the lead vehicle stops first, the terminal displacements of the vehicles determine their minimum range. In the lower half of the figure, the following vehicle stops first and appears to stop just short of the lead vehicle’s final position. However, the minimum separation occurs before the vehicles are at rest, and, indeed, the displacement curves in the figure overlap, indicating 0 minimum separation, i.e., a collision. When does this minimum separation occur? The minimum separation occurs at the precise instant that the following and lead vehicles have the same velocity. Before this, the following vehicle traveling faster than the lead vehicle; afterwards, it is traveling slower. In conclusion:

When the lead vehicle stops first, only the terminal displacements of the vehicles matter.

When the following vehicle stops first, the minimum separation of the vehicles occurs when the following and lead vehicles have the same speed.

Determining which analysis applies. Heuristically, long reaction times and/or modest following vehicle deceleration levels will result in the lead vehicle stopping at some time before the following vehicle. Conversely, a combination of a fast reaction and high level of deceleration will result in the following vehicle stopping before the lead vehicle. The lead vehicle stops at time

$$t = -\frac{v_L}{a_L},$$

and the following vehicle stops at time

$$t = t_r - \frac{v_F}{a_F}.$$

For the terminal location analysis above to hold, we require the lead vehicle to stop at a time at or before the following vehicle stops. Mathematically, we require collision

$$-\frac{v_L}{a_L} \leq t_r - \frac{v_F}{a_F}.$$

Rearranging this equation yields a simple linear relation between the reaction time and the following vehicle acceleration:

$$\frac{1}{a_F} v_F - \frac{v_L}{a_L} \leq t_r, \quad (4)$$

which states the required condition for the application of the terminal location analysis. Two limit cases result from the relation in (4). Were infinite deceleration of the following vehicle somehow possible, the maximum reaction time would be provided by $-v_L/a_L$. This would result in both vehicles stopping at the same time. For zero reaction time, a following vehicle acceleration of $v_F a_L / v_L$ would also result in both vehicles stopping at the same time.

The analysis relation in (4) and its interpretation are shown in Figure 4. As the figure indicates, high levels of following vehicle deceleration and quick reactions (points near the origin) correspond to the following vehicle stopping before the the lead vehicle and the need to apply a minimum separation analysis. Conversely, low deceleration levels or slow reactions (points away from the origin) correspond to the lead vehicle stopping first and the need to apply a terminal location analysis.

The development in this section does not indicate if a collision will occur. That is, a lead vehicle stopping before a following vehicle, or vice versa, implies nothing regarding a collision (or lack thereof) between the vehicles. The stopping sequence only determines which analysis should be applied, and the plot in Figure 4 indicates how to determine which analysis should be used.

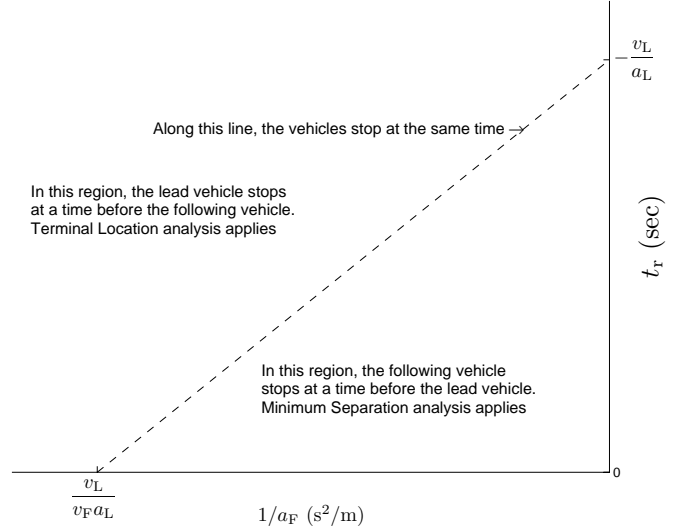


Figure 4: Terminal-condition and minimum-separation analyses boundary line

The next two sections develop the terminal location and minimum separation analyses.

Terminal location analysis. The terminal locations of the following and lead vehicles suffice to analyze the envelope of opportunity when the lead vehicle stops before the following vehicle. To avoid a collision, the initial range, R , plus the braking distance of the lead vehicle must be greater than or equal to the distance the following vehicle travels before braking plus the distance it travels after braking. Expressing the equality case mathematically,

$$R + \Delta R_1 = \Delta R_{21} + \Delta R_{22}, \text{ where} \quad (5)$$

$$\Delta R_1 = -\frac{v_L^2}{2a_L}$$

$$\Delta R_{21} = v_F \times t_r$$

$$\Delta R_{22} = -\frac{v_F^2}{2a_F}.$$

Substitution of these relations into (5) yields

$$R - \frac{v_L^2}{2a_L} = v_F t_r - \frac{v_F^2}{2a_F}. \quad (6)$$

Solving (6) for t_r leads to

$$t_r = \frac{v_F}{2} \frac{1}{a_F} + \frac{R - \frac{v_L^2}{2a_L}}{v_F}. \quad (7)$$

Two limits result from this relation:

$$\lim_{t_r \rightarrow 0} \frac{1}{a_F} = \frac{1}{v_F^2} \left(\frac{v_L^2}{a_L} - 2R \right) \quad (8a)$$

$$\lim_{a_F \rightarrow -\infty} t_r = \frac{1}{v_F} \left(R - \frac{v_L^2}{2a_L} \right) \quad (8b)$$

The plot of reaction time versus the reciprocal of the following vehicle acceleration is the same as that shown in Figure 2, but the zero-reaction-time acceleration and time-to-collision change to the limits provided in (8).

Minimum separation analysis. This section develops analysis relations for the case in which the following vehicle comes to rest before the lead vehicle. As indicated, the minimum separation of the vehicles will occur when both vehicles are in motion. To avoid a collision, the initial range plus the subsequent displacement of the lead vehicle must always be greater than the displacement of the following vehicle. Mathematically, the following inequality must hold until the lead vehicle stops

$$R + v_L t + \frac{a_L t^2}{2} \geq \begin{cases} v_F t & \text{if } 0 \leq t \leq t_r \\ v_F t + \frac{a_F (t - t_r)^2}{2} & \text{if } t_r < t \leq t_r - \frac{v_F}{a_F} \end{cases} \quad (9)$$

The maximum reaction time of the following vehicle, t_{r_max} , is examined first. This is the time-to-collision when the following vehicle does not react to a decelerating lead vehicle. This time equals the positive root of the following quadratic, where $a_L < 0$ is assumed:

$$R + v_L t + \frac{1}{2} a_L t^2 - v_F t = 0. \quad (10)$$

The positive root of this equation is

$$t_{r_max} = \frac{v_F - v_L - \sqrt{(v_L - v_F)^2 - 2Ra_L}}{a_L}, \quad (11)$$

which provides a convenient upper-bound for the reaction time in subsequent analyses.

Assuming that the following vehicle brakes before the maximum reaction time, the resulting envelope of opportunity available to the driver is of interest. The case in which the following vehicle stops at or before the time the lead vehicle stops is under discussion, and the envelope of opportunity is based on the minimum separation of the two vehicles. Although many approaches are available for this analysis, the most direct is to (1) employ a relative coordinate frame in formulating the kinematics problem and (2) formulate the problem *at* the reaction time. With the usual initial conditions and the assumption that $a_F = 0$ for $t < t_r$, the following kinematic

variables result at $t = t_r$:

$$\begin{aligned} R(t_r) &= R + (v_L - v_F)t_r + \frac{a_L t_r^2}{2} \\ v_L(t_r) - v_F &= v_L + a_L t_r - v_F \\ a_{L/F}(t_r) &= a_L - a_F. \end{aligned}$$

The relative change in displacement in the interval between t_r and the time at which the vehicles have the same speed (when separation is minimum) equals

$$\Delta R = -\frac{(v_L(t_r) - v_F)^2}{2(a_L - a_F)},$$

where $v_F \geq v_L(t_r)$ is assumed. To avoid a collision, the original displacement at the reaction time plus the relative change in displacement must be positive. In the limit case, in which the vehicles minimum separation equals 0,

$$R(t_r) - \frac{(v_L(t_r) - v_F)^2}{2(a_L - a_F)} = 0$$

holds. Solving for a_F results in

$$a_F = a_L - \frac{(v_L(t_r) - v_F)^2}{2R(t_r)}, \quad (12)$$

which defines the minimum-separation envelope of opportunity acceleration as a function of reaction time. One limit results from this relation:

$$\lim_{t_r \rightarrow 0} a_F = a_L - \frac{(v_L - v_F)^2}{2R}, \quad (13)$$

which provides the acceleration axis intercept of the envelope of opportunity. The reaction time intercept, the maximum reaction time, is provided by (11). Equation (11) also provides a check for (12). Recall that (11) assumes no following vehicle deceleration and that its solution provides the time to collision, i.e., the time at which the range equals 0. The substitution of $R(t_r) = 0$ into (12) results in $a_F = -\infty$, which corresponds to the required deceleration for a following vehicle initially braking at $t = t_{r_max}$.

By substituting values of t_r in the range 0 – t_{r_max} into (12) we can determine boundary points for the envelope of opportunity. While the minimum separation analysis does not yield this envelope as directly as the zero-lead-vehicle acceleration or the terminal location cases, the relation is simple to apply and makes intuitive sense. To avoid a collision, the difference in acceleration between the following and lead vehicles is proportional to the range rate squared when the warning is issued and inversely proportional to the range, which is the same information provided in (3b).

Example. An example with simple parameters will cohere the concepts developed in this section. A decelerating lead vehicle scenario has the following initial conditions:

$$v_L = 2, a_L = -1, v_F = 4, R = 4.$$

First, the “analysis boundary” that separates the terminal location and minimum separation analyses is determined. Using (4), the following end-points of this straight-line boundary result:

$$\begin{aligned} \lim_{t_r \rightarrow 0} a_F &= \frac{v_L}{v_F a_L} = -0.5 \\ \lim_{a_F \rightarrow -\infty} t_r &= -\frac{v_L}{a_L} = 2. \end{aligned}$$

Next, the end points of the straight-line terminal location envelope of opportunity are obtained using (8):

$$\begin{aligned} \lim_{t_r \rightarrow 0} a_F &= \frac{1}{v_F^2} \left(\frac{v_L^2}{a_L} - 2R \right) = -0.75 \\ \lim_{a_F \rightarrow -\infty} t_r &= \frac{1}{v_F} \left(R - \frac{v_L^2}{2a_L} \right) = 1.50 \end{aligned}$$

A series of points for the minimum separation envelope of opportunity are needed next. The end points for this envelope are found using (11) and (12):

$$\begin{aligned} \lim_{a_F \rightarrow -\infty} t_r &= \frac{v_F - v_L - \sqrt{(v_L - v_F)^2 - 2Ra_L}}{a_L} = 1.4641 \\ \lim_{t_r \rightarrow 0} a_F &= \left(a_L - \frac{(v_L - v_F)^2}{2R} \right)^{-1} = -0.6667 \end{aligned}$$

Intermediate points are synthesized using (12), while varying t_r from 0 to 1.4641.

Plotting the terminal location and the minimum separation envelopes of opportunity and the analyses-boundary on the same axes provides a comprehensive understanding of the envelope of opportunity available with the initial conditions provided above. These are shown in Figure 5. Figure 5 indicates that the minimum separation envelope of opportunity is tighter than the terminal location envelope. This is precisely what is expected, since the minimum separation condition is a more restrictive condition. Further, these two envelopes intersect at the analysis boundary, which is again anticipated. Along the analysis boundary, the kinematic initial conditions are such that the following and lead vehicles will both come to a stop at the same time. At the mutual intersection point of the two envelopes and the analysis boundary, the vehicles not only come to rest at the same time, they also come to rest at the same location.

The net envelope of opportunity is a single curve composed of: (1) the portion of the minimum separation envelope to the right of the

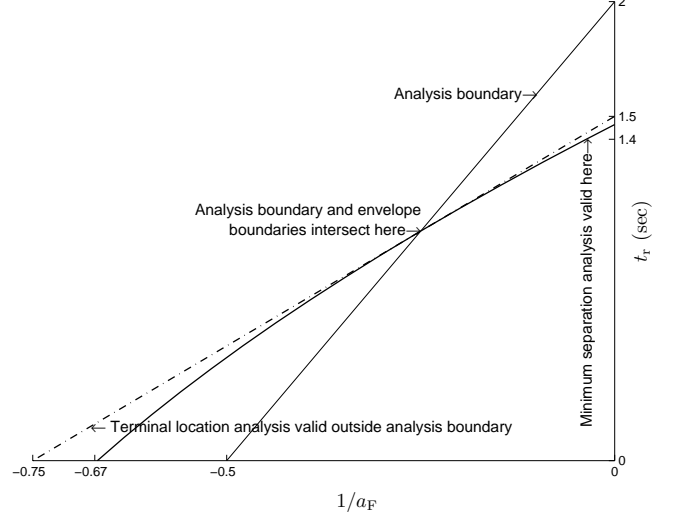


Figure 5: Envelopes of opportunity and analysis-boundary for $v_L = 2, a_L = -1, v_F = 4, R = 4$

intersection point in Figure 5 and (2) the portion of the terminal condition envelope to the left of the intersection point.

4 ENVELOPES OF OPPORTUNITY PROVIDED BY TWO ALGORITHMS

In this section we use the relations developed in the previous section to analyze the performance of two warning algorithms. The first of these algorithms, a hypothetical *headway algorithm*, relies on simple processing of kinematic variables to determine the need to issue a warning. The other algorithm, a *second-order algorithm*, uses second-order kinematics to evaluate the need to issue alerts [5].

The CAMP study recommends some 17 rear-end collision scenarios for evaluating a collision warning system [8, chapter 5]. These scenarios include

- C-4** The test consists of a subject vehicle traveling at 100 kph approaching a lead vehicle that is parked under an overhead sign.
- C-3** A subject vehicle follows a lead vehicle, which is initially traveling at the same speed as the subject vehicle. The subject vehicle is following at a moderate distance when the lead vehicle begins to brake moderately hard. The subject vehicle maintains a constant speed until the required crash alert is triggered.

We will examine the performance of two collision warning algorithms with these two scenarios.

4.1 Stopped Lead Vehicle

For a stopped lead vehicle, the headway algorithm issues an alert when the subject vehicle is within 3 seconds of the stopped vehicle, i.e., when $-R/\dot{R} = 3$. The second-order algorithm issues its warning based on an assumed reaction time of 1.5 seconds and a braking level of -0.75 g for the subject vehicle. Furthermore, it includes a second-order term for the subject vehicle velocity. With this assumption the warning distance is

$$R_w = -\frac{v_F^2}{2a_F} + 1.5v_F + 2, \quad (14)$$

where v_F and a_F are given in SI units. For the C-4 scenario above, the algorithms issue the following alerts

headway: $R_w = 83.3$ m

second-order: $R_w = 100.8$ m.

For each case $\dot{R} = -v_F = -27.78$ m/s. Plots of the respective envelopes of opportunity, as calculated using these alerts, are shown in Figure 6.

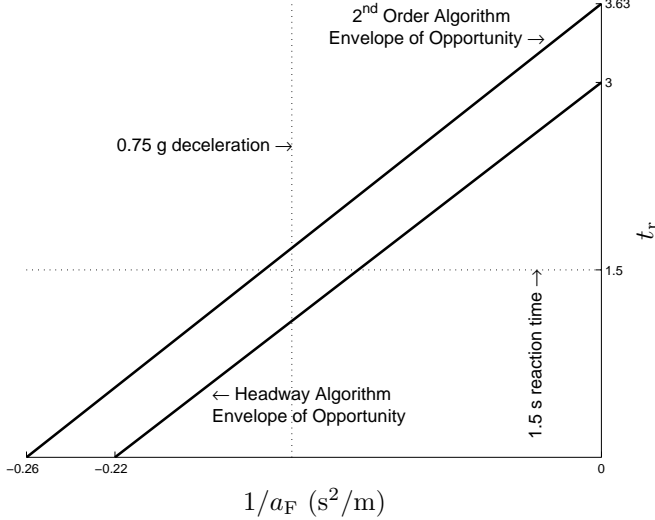


Figure 6: Envelopes of opportunity for stopped vehicle using two collision warning algorithms

The shorter solid line in Figure 6 bounds to the envelope of opportunity provided by the 3 second headway algorithm. With an assumed 1.5 second reaction time and 0.75 g deceleration, the analysis indicates that the subject vehicle would collide with the lead vehicle. The longer solid line in the same figure passes slightly above and to the left of the intersection of a 1.5 second reaction time and 0.75 g deceleration. With the same assumptions, the analysis indicates that the subject would stop

behind the lead vehicle with a slight margin, which is exactly what the warning algorithm intends.

4.2 Decelerating Lead Vehicle

For this test the following and lead vehicles are initially traveling at 100 kph, with a 1-second headway between them, and the lead vehicle decelerates at 0.32 g. The second-order algorithm issues an alert at the following warning time

$$t_w = \frac{0.75 + \frac{a_L}{9.81}}{0.75} \left(\frac{2(v_0 t_h - 2)}{-a_L \left(1 + \frac{a_L}{9.81 \times 0.75} \right)} \right)^{0.5} - 1.5 = 1.57 \text{ sec}, \quad (15)$$

when $a_L = -0.32 \times 9.81$. The corresponding warning distance is

$$R_w = R_0 + \frac{1}{2} a_L t_w^2 = 23.9 \text{ m},$$

and the corresponding range rate when the alert is issued equals -4.9 m/s. The following vehicle velocity plus this range rate equals the lead vehicle velocity. These values and the lead vehicle acceleration parameterize (8b) and (8a) to obtain the end points of the envelope of opportunity line.

The headway algorithm is assumed to allow a 1-second following distance, provided that the range rate is non-negative. An alert is issued as soon as the lead vehicle begins to decelerate. In this case the initial range of 27.78 m, a common velocity of 27.78 m/s, and the lead vehicle acceleration parameterize (8b) and (8a), which determine the envelope of opportunity line.

The envelopes of opportunity using the second-order and 1-second headway algorithms are shown in Figure 7. In this figure, by combining the applicable portions of each envelope, the envelopes of opportunity obtained using the two algorithms have been shown as single curves,

In Figure 7 the envelope of opportunity provided by the second-order algorithm just avoids penetrating the rectangle bounded by a 1.5 second reaction time and a 0.75 g level of deceleration, just as the algorithm intends. The one-second headway algorithm avoids the rectangle by a large margin. While one might argue that the large response margin is safer than a tight margin, some drivers may perceive the warning as a nuisance warning. A high frequency of (perceived) nuisance alerts will likely result in a driver ultimately ignoring the alerts, effectively canceling the vigilance provided by the warning system.

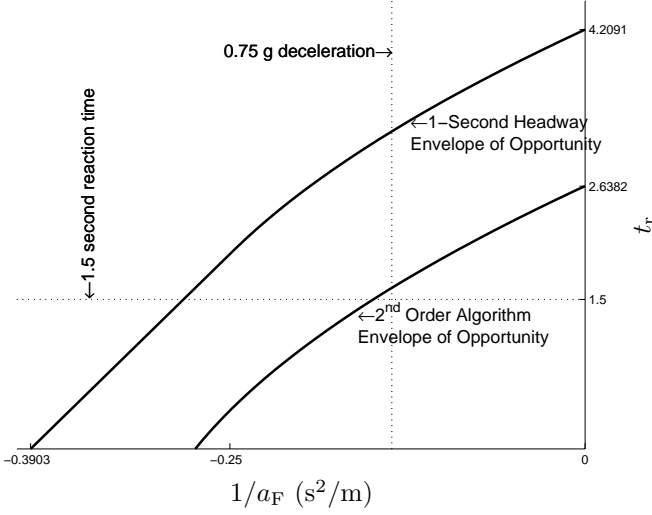


Figure 7: Envelopes of opportunity for decelerating lead vehicle using two collision warning algorithms

5 DISCUSSION

This paper analyzes the envelope of opportunity available to a following-vehicle driver who has received an alert regarding an impending collision with a lead vehicle. The envelope is depicted in a coordinate plane, with the vertical axis representing reaction time and the horizontal axis representing the reciprocal of the following vehicle acceleration. The three scenarios addressed—slowing, slow, or stopped lead vehicle—comprise some 90% of all rear-end collisions [9].

The methods developed provide a ready assessment of the envelope of opportunity. At the instant an alert is issued, the values of certain kinematic variables are noted. These values are then substituted into simple algebraic relations. The relations provide the end points for the following

straight-line envelope that applies in cases of constant speed lead vehicles and the lead vehicle coming to a stop before the following vehicle;

curved envelope that applies when the following vehicle comes to a stop before the lead vehicle; and

straight-line boundary that indicates which of the above envelopes applies in the case of a decelerating lead vehicle.

The analyses provided here complement previous work in this area. The envelope of opportunity when the lead vehicle is stationary is addressed [4]. Warning algorithms are developed for constant velocity and decelerating lead vehicles in [5]. The current paper begins with the constant velocity lead vehicle, plotting the envelope

using the reciprocal of the following vehicle acceleration as the horizontal axis coordinate. The use of the reciprocal of acceleration, in this and several other cases, leads to a linear relationship between the required reaction time and the required acceleration. Furthermore, the limiting case of infinite following vehicle deceleration is easily included in the plot, which provides the time-to-collision variable. Although [5] considers the case of a decelerating lead vehicle, the work reported here attempts to clarify—using symbolic variables and distinct plots—when the terminal location and minimum separation analyses apply. This is performed for the general case when $v_F \geq v_L$ and $a_L < 0$.

In [6] Burgett et al. review adaptive cruise control field operational test results and discuss levels of deceleration “authority” for ACC-equipped vehicles. The paper provides a range (R) versus range rate (\dot{R}) plot that illustrates the relation between following and lead vehicle deceleration authorities and (R, \dot{R}) pairs that these will accommodate, i.e., decelerate a following vehicle sufficiently to avoid a collision. Each following-lead vehicle deceleration authority pair requires its own curve on the $R\text{-}\dot{R}$ plot. Although not explored in the present paper, the envelope of opportunity plots could also find application in adaptive cruise control evaluation, given that such systems themselves have reaction times and deceleration levels.

In addition to providing the various envelopes of opportunity, the analysis methods used in this paper may have a broader application. The geometric approach used to compute absolute and relative changes in displacements will likely simplify the envelope analyses for both partially decelerating lead vehicles and accelerating following vehicles, cases that will need to be considered when evaluating a rear-end collision warning system. Furthermore, in Section 3.2 the minimum-separation analysis is simplified by treating the range and lead vehicle velocity at the reaction time (t_r) as *initial conditions* for the kinematics problem. The alternative approach of carrying the reaction time term in the analysis and solving for the reaction time as a function of the remaining kinematic variables produces a large, multiterm quadratic equation that provides little insight into the dominant parameters governing the envelope of opportunity.

The initial-condition approach used in the paper provides a very insightful relation for the required following vehicle acceleration (12), which we repeat here:

$$a_F = a_L - \frac{(v_L(t_r) - v_F)^2}{2R(t_r)}.$$

For a decelerating lead vehicle, as the reaction time increases the term $v_L(t_r) - v_F$ will increase linearly with the reaction time and the term $R(t_r)$ decreases with the

square of the reaction time. The magnitude of the following vehicle acceleration that is necessary to avoid a collision will thus increase with the square of the reaction time in a compound sense: the numerator increases with reaction time squared and the denominator decreases with reaction time squared.

Developers and evaluators of collision warning algorithms require various tools to analyze algorithm performance. Such algorithms alert a driver of an impending collision and provide a driver with a certain envelope of opportunity to avoid a collision. The “size” of this envelope is highly dependent on the timing of the alert. While algebraic analysis provides one tool for algorithm development and assessment, a geometric approach that depicts the size of the envelope of opportunity will convey essential information to the analyst as or more readily than algebraic equations. Our future efforts will include using the methods provided here to evaluate the data obtained through pilot testing of a truck with a collision warning system and field operational testing of a light-vehicle rear-end collision warning system.

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